

ACE II – Mr. Bo  
Final Review #1 – Answers

<b>1&gt;</b> $\frac{d}{dx} \ln(\sqrt{x^2 + 1}) = \frac{x}{x^2 + 1}$	<b>14&gt;</b> $\frac{d}{dx} (x^2 2^x) = 2x 2^x + x^2 2^x \ln(2)$
<b>2&gt;</b> $\frac{d}{dx} \log_3(x^2) = \frac{2}{x \ln(3)}$	<b>15&gt;</b> $\int \frac{x}{\sqrt{1 - x^4}} dx = \frac{1}{2} \arcsin(x^2)$
<b>3&gt;</b> $\frac{d}{dx} e^{x^2 + 2} = 2x e^{x^2 + 2}$	<b>16&gt;</b> $\int \frac{1}{x \ln(x)} dx = \ln(\ln(x))$
<b>4&gt;</b> $\frac{d}{dx} 3^{\ln(x)} = \frac{3^{\ln(x)} \ln(3)}{x}$	<b>17&gt;</b> $\int \frac{e^{2x}}{1 + e^{2x}} dx = \frac{1}{2} \ln(1 + e^{2x})$
<b>5&gt;</b> $\int \frac{x}{x^2 + 3} dx = \frac{1}{2} \ln(x^2 + 3)$	<b>18&gt;</b> $\frac{d}{dx} \operatorname{sech}(x + 1) = -\operatorname{sech}(x + 1) \tanh(x + 1)$
<b>6&gt;</b> $\int e^{3x} dx = \frac{1}{3} e^{3x}$	<b>19&gt;</b> $\frac{d}{dx} (x^2 e^{-x}) = 2x e^{-x} - x^2 e^{-x}$
<b>7&gt;</b> $\int \frac{1}{x} 3^{\ln(x)} dx = \frac{3^{\ln(x)}}{\ln(3)}$	<b>20&gt;</b> $\int \frac{(\csc(x))^2}{\cot(x)} dx = -\ln( \cot(x) )$
<b>8&gt;</b> $\int \sec(4x) dx = \frac{1}{4} \ln(\sec(4x) + \tan(4x))$	<b>21&gt;</b> $\frac{d}{dx} \ln(\ln(x)) = \frac{1}{x \ln(x)}$
<b>9&gt;</b> $\frac{d}{dx} \arcsin(4x^2) = \frac{8x}{\sqrt{1 - 16x^4}}$	<b>22&gt;</b> $\int \frac{x - 3}{x^2 + 1} dx = \frac{1}{2} \ln(x^2 + 1) - 3 \arctan(x)$
<b>10&gt;</b> $\frac{d}{dx} \arctan(e^x) = \frac{e^x}{1 + (e^x)^2}$	<b>23&gt;</b> $\int \frac{3^{2x}}{1 + 3^{2x}} dx = \frac{1}{2} \frac{\ln(1 + 3^{2x})}{\ln(3)}$
<b>11&gt;</b> $\frac{d}{dx} \operatorname{arcsec}(x^2) = \frac{2}{x\sqrt{x^4 - 1}}$	<b>24&gt;</b> $\int \frac{\arcsin(x)}{\sqrt{1 - x^2}} dx = \frac{1}{2} \arcsin(x)^2$
<b>12&gt;</b> $\int \frac{x + 2}{3 + 4x^2} dx = \frac{1}{8} \ln(3 + 4x^2) + \frac{1}{3} \sqrt{3} \arctan\left(\frac{2}{3} x \sqrt{3}\right)$	<b>25&gt;</b> $\frac{d}{dx} \operatorname{arccot}(\sqrt{x}) = -\frac{1}{2\sqrt{x}(x + 1)}$
<b>13&gt;</b> $\frac{d}{dx} \cosh(4x^2) = 8 \sinh(4x^2) x$	

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Final Review #2 – Answers

$$1> \frac{d}{dx} (x\sqrt{x^2-3}) = \sqrt{x^2-3} + \frac{x^2}{\sqrt{x^2-3}}$$

$$2> \int \frac{x^3 - 3x^2 + 5}{x-3} dx = \frac{1}{3}x^3 + 5\ln(x-3)$$

$$3> \int \frac{1}{x^2 + 4x + 13} dx = \frac{1}{3} \arctan\left(\frac{1}{3}x + \frac{2}{3}\right)$$

$$4> \int \frac{3}{x^2 + x - 2} dx = -\ln(x+2) + \ln(x-1)$$

$$5> \int (x^4 \ln(x)) dx = \frac{1}{5}x^5 \ln(x) - \frac{1}{25}x^5$$

$$6> \int (1 - \sin(x)^2) \sin(x)^4 \cos(x) dx = -\frac{1}{7} \sin(x)^7 + \frac{1}{5} \sin(x)^5$$

$$7> \int \frac{\sqrt{25-x^2}}{x} dx = -5 \ln\left(\left|\frac{5 + \sqrt{25-x^2}}{x}\right|\right) + \sqrt{25-x^2}$$

$$8> \int (x^2 \cos(x)) dx = x^2 \sin(x) - 2 \sin(x) + 2x \cos(x)$$

$$9> \int \frac{x+2}{x^2-4x} dx = -\frac{1}{2} \ln(x) + \frac{3}{2} \ln(x-4)$$

$$10> \int \frac{x+5}{\sqrt{9-(x-3)^2}} dx = -\sqrt{-x^2+6x} + 8 \arcsin\left(\frac{1}{3}x-1\right)$$

$$11> \frac{d}{dx} \cosh(x^2+x) = \sinh(x^2+x) (2x+1)$$

$$12> \int \frac{5}{\sqrt{9-x^2}} dx = 5 \arcsin\left(\frac{1}{3}x\right)$$

$$13> \int \frac{2x-5}{x^2+2x+2} dx = \ln(x^2+2x+2) - 7 \arctan(x+1)$$

$$14> \int \frac{9x^3}{\sqrt{1+x^2}} dx = 3\sqrt{(1+x^2)^3} - 9\sqrt{1+x^2}$$

$$15> \int \tan(x)^2 \sec(x)^2 dx = \frac{1}{3} \tan^3 x$$

$$16> \frac{d}{dx} \frac{x^2}{(x^3-2)^2} = \frac{2x}{(x^3-2)^2} - \frac{6x^4}{(x^3-2)^3}$$

$$17> \int \frac{x}{2} (1 - \cos(2x)) dx = \frac{1}{4} x^2 - \frac{1}{8} \cos(2x) - \frac{1}{4} x \sin(2x)$$

$$18> \frac{d}{dx} \operatorname{arcsec}(2x) = \frac{2}{2|x|\sqrt{4x^2-1}}$$

$$19> \int \frac{1}{2} (1 + \cos(6x)) dx = \frac{1}{2} x + \frac{1}{12} \sin(6x)$$

$$20> \frac{d}{dx} x^{(x-1)} = x^{x-1} \left( \ln(x) + \frac{x-1}{x} \right)$$

$$21> \int \frac{2x^2 + 7x - 3}{x-2} dx = x^2 + 11x + 19 \ln(x-2)$$

$$22> \frac{d}{dx} \log_5(\sqrt{x^2-1}) = \frac{x}{(x^2-1) \ln(5)}$$

$$23> \int \sec(x)^3 dx = \frac{1}{2} [\ln(|\sec(x) + \tan(x)|) + \sec(x)\tan(x)]$$

ACE Calculus II – Mr. Bo  
Final Review #3 – Answers

1> $\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x - 2} = 3$	14. $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^3 - 1} \Rightarrow \text{converge absolutely}$
2> $\int_1^{\infty} \frac{1}{x^2} dx = 1$	15> $\int_1^{\infty} \frac{\ln(x)}{x} dx = \infty \Rightarrow \text{diverges}$
3. $a_n = \frac{(n-2)!}{n!} \Rightarrow \text{converges to 0}$	16> $\lim_{x \rightarrow \infty} \frac{5x^2 - 3x + 1}{3x^2 - 4} = \frac{5}{3}$
4> $\sum_{n=1}^{\infty} \frac{3n-1}{2n+1} \Rightarrow \text{diverges}$	17. $\sum_{n=1}^{\infty} \frac{(n!)^2}{(3n)!} \Rightarrow \text{converges}$
5. $\sum_{n=1}^{\infty} 4\left(\frac{1}{2}\right)^n \Rightarrow \text{converges}$	18> $\int \frac{\sqrt{1-x^2}}{x^4} dx = -\frac{1}{3} \frac{(1-x^2)^{3/2}}{x^3}$
6. $\sum_{n=1}^{\infty} \left(\frac{1}{n} + \frac{1}{n+2}\right) \Rightarrow \text{converges}$	19> $\lim_{x \rightarrow \infty} \frac{\ln(x)^2}{x^3} = 0$
7. $\sum_{n=1}^{\infty} \frac{4}{4\sqrt{n^3}} \Rightarrow \text{diverges}$	20. $\sum_{n=1}^{\infty} \frac{3^n}{n^2} \Rightarrow \text{diverges}$
8> $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} \Rightarrow \text{converges}$	21> $\int_0^1 \frac{1}{\sqrt[3]{(x-1)^2}} dx = 3$
9> $\sum_{n=1}^{\infty} \frac{1}{n^2 + 1} \Rightarrow \text{converges}$	22> $\int \sin(x) (1 - \cos(x)^2) \cos(x)^4 dx = \frac{1}{7} \cos(x)^7 - \frac{1}{5} \cos(x)^5$
10. $\sum_{n=1}^{\infty} \frac{n!}{n3^n} \Rightarrow \text{diverges}$	23. $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n+4}} \Rightarrow \text{converges conditionally}$
11. $\sum_{n=1}^{\infty} \frac{(-1)^n}{(\ln n)^n} \Rightarrow \text{converges}$	24> $\frac{d}{dx} \left( 3 \arccos\left(\frac{x}{2}\right) \right) = -\frac{3}{\sqrt{4-x^2}}$
12. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n-1}} \Rightarrow \text{diverges}$	25. $a_n = \frac{3n^2(2n)!}{5(2n+2)!} \Rightarrow \text{converges to } \frac{3}{5}$
13. $\sum_{n=1}^{\infty} \frac{2}{3^n - 5} \Rightarrow \text{converge}$	

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Final Review #4 – Answers

<p><b>1&gt;</b> <math>\ln(x+1) \xrightarrow{\text{series in } x} x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4</math></p>	<p>15. <math>\sum_{n=0}^{\infty} (2n)! \left(\frac{x}{2}\right)^n \Rightarrow R=0</math></p>
<p><b>2&gt;</b> <math>\frac{3}{x+2} = \sum_{n=1}^{\infty} \frac{3(-1)^n}{2^{n+1}} x^n</math></p>	<p><b>16&gt;</b> <math>y' = \sqrt{\left(\frac{x^2-1}{x^2+1}\right)} \left[ \frac{x}{x^2-1} - \frac{x}{x^2+1} \right]</math></p>
<p>3. <math>\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n} x^n \Rightarrow R=2</math></p>	<p><b>17&gt;</b> <math>\frac{d}{dx} \log_3(x^4+1) = \frac{4x^3}{(1+x^4)\ln(3)}</math></p>
<p>4. <math>\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(n+1)(n+2)}</math> converges on <math>-1 \leq x \leq 1</math></p>	<p>18. <math>\sum_{n=1}^{\infty} \frac{(-1)^n 5^n}{8^n} \Rightarrow</math> converges absolutely</p>
<p>5. <math>e^{3x+2} = e^2 \left( 1 + (3x) + \frac{(3x)^2}{2!} + \frac{(3x)^3}{3!} + \dots + \frac{(3x)^n}{n!} + \dots \right)</math></p>	<p><b>19&gt;</b> <math>\lim_{x \rightarrow \infty} x^{\frac{1}{x}} = 1</math></p>
<p>6. <math>\sum_{n=0}^{\infty} \frac{(3x)^n}{(2n)!} \Rightarrow R=\infty</math></p>	<p>20. <math>\sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x-2)^n}{n2^n} \Rightarrow</math> converges on <math>0 &lt; x \leq 4</math></p>
<p>7. <math>\sum_{n=0}^{\infty} \frac{4^n}{3^n+1} \Rightarrow</math> diverges</p>	<p><b>21&gt;</b> <math>\int \frac{5^x}{1+5^{x+1}} dx = \frac{1}{5} \frac{\ln(1+5^{x+1})}{\ln(5)}</math></p>
<p>8. <math>\frac{4}{3x+2} = \sum_{n=0}^{\infty} \frac{(-1)^n (3)^n}{2(8)^n} (x-2)^n</math></p>	
<p>9. <math>\sum_{n=1}^{\infty} \frac{5n-3}{n^2-2n+5} \Rightarrow</math> diverges</p>	
<p><b>10&gt;</b> <math>\frac{d}{dx} \operatorname{arccot}(x^2) = -\frac{2x}{1+x^4}</math></p>	
<p><b>11&gt;</b> <math>\sqrt{x} \xrightarrow{\text{series in } x} \frac{1}{2} + \frac{1}{2}x - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3 - \frac{5}{128}(x-1)^4</math></p>	
<p><b>12&gt;</b> <math>\int \sqrt{4+9x^2} dx = \frac{2}{3} \left[ \ln \left( \left  \frac{1}{2} \sqrt{4+9x^2} + \frac{3}{2}x \right  \right) + \frac{3}{4}x\sqrt{4+9x^2} \right]</math></p>	
<p><b>13&gt;</b> <math>\int_0^1 \frac{1}{x^2} dx = \infty</math></p>	
<p>14. <math>\sin(\pi x) = (\pi x) - \frac{(\pi x)^3}{3!} + \frac{(\pi x)^5}{5!} - \frac{(\pi x)^7}{7!} + \dots + \frac{(-1)^n (\pi x)^{2n+1}}{(2n+1)!} + \dots</math></p>	

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Final Review #5 – Answers

1> $y^3 = \frac{3}{2}(t^2 + 1) - \frac{9}{2}$	14. $P_3(x) = \ln(2) + \frac{1}{2}(x-2) - \frac{1}{4(2!)}(x-2)^2 + \frac{1}{4(3!)}(x-2)^3$
2> $y = \left(\frac{1}{4}x\right)^2 + 1$	15> $\frac{d}{dx}[\tanh(\sqrt{x-4})] = \frac{1}{2\sqrt{x-4}}\operatorname{sech}^2(\sqrt{x-4})$
3. $\frac{dy}{dx} = 1 \quad \frac{d^2y}{dx^2} = 2$	16> $(x-2)^2 + \frac{y^2}{9} = 1$
4> $\int_0^2 2\sqrt{t^2+1} dt \approx 5.92$	17. $\int x \ln(x+1) dx = \frac{1}{2}x^2 \ln(x+1) - \frac{1}{2}\ln(x+1) - \frac{1}{4}x^2 - \frac{1}{2}x + c$
5. $r = \frac{-2}{3\cos\theta - \sin\theta}$	18> $\frac{1}{2} \int_0^{\pi/2} 36\sin^2(2\theta) d\theta = \frac{9\pi}{2}$
6. $x^2 + y^2 - 5x = 0$	19> $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$
7. $x = (2 + \sin\theta)\cos\theta$ $y = (2 + \sin\theta)\sin\theta$	20. $\sum_{n=0}^{\infty} \left(\frac{-1}{5}\right) \left(\frac{2}{5}x\right)^n$
8> $\frac{dy}{dx} = -\sqrt{3}$	21> $\lim_{x \rightarrow \infty} x \tan\left(\frac{1}{x}\right) = 1$
9> $\int_0^{\pi/10} 25\cos^2(5\theta) d\theta = \frac{5\pi}{4}$	
10. $\int_1^2 \frac{1}{\sqrt{4-x^2}} dx = \frac{\pi}{3}$	
11. $t = \pm 1 \Rightarrow (0, -2), (2, 2)$	
12. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{2n-1}$ diverges	
13. $\int e^x \sqrt{1-e^x} dx = \frac{-2}{3}(1-e^x)^{5/2} + c$	